Quadratic Equations & Functions (QUA), Part 1

In this unit, you will learn how to:

- Use factoring to answer quadratics questions.
- Work with quadratic equations in standard form.
- Use the quadratic formula and complete the square to solve quadratic equations.

Introduction to Quadratic Functions

A quadratic function is a polynomial of degree 2, meaning the highest power of the variable is 2. For example, the following equations represent quadratic functions:

$$f(x) = 5x^2 - 2x + 9 \qquad g(x) =- x^2 \qquad h(x) = (x - 5)(x + 2)$$

The **graph of a quadratic function is a** "U"-shaped figure called a **parabola**.

Practice Question

1

$$g(x) = x^2 + 8$$

Which table gives three values of x and their corresponding values of $g(x)$ for the given function g?

A)

x	$g(x)$
1	9
2	10
3	11

B)

x	$g(x)$
1	9
2	12
3	17

C)

x	$g(x)$
1	-7
2	12
3	17

D)

x	$g(x)$
1	9
2	12
3	11

Factoring

There are situations where factoring—not the quadratic formula—is the best method. Let's take a look at some key factoring skills.

Factoring Quadratics in the Form $x^2 + bx + c$

You will often be asked to factor a standard form quadratic function in the form $f(x) = x^2 + bx + c$ (in other words, one where $a = 1$). In such cases, the function when factored will equal $(x + j)(x + k)$, where $j + k = b$ and $jk = c$. Consider the example below.

Example

Which expression is equivalent to $x^2 + 3x - 28$?

A. $(x - 2)(x + 14)$

B. $(x - 4)(x + 7)$

C. $(x + 2)(x - 14)$

D. $(x + 4)(x - 7)$

Because $-4 + 7 = 3$ and $-4 \bullet 7 = -28$, the correct answer is B. We can confirm this using distribution: $(x - 4)(x + 7) = x^2 + 7x - 4x - 28 = x^2 + 3x - 28$. Remember that you can also check your answer by graphing $y = x^2 + 3x - 28$ and $y = (x - 4)(x + 7)$ and making sure they represent the same graph.

Practice Questions

2

Which expression is equivalent to $x^2 + 4x - 32$?

A) $(x - 2)(x + 16)$

B) $(x - 4)(x + 8)$

C) $(x - 8)(x + 4)$

D) $(x - 16)(x + 2)$

3

$$x^2 - 4x - 12$$

The expression above is equivalent to $(x + r)(x + w)$, where r and w are constants. What is the value of $|r - w|$?

You can also use factoring to solve equations. Consider the example below:

Example

$x^2 + 3x - 28 = 0$. What is a possible value of x?

Remember that $x^2 + 3x - 28 = (x - 4)(x + 7)$. This means that $(x - 4)(x + 7) = 0$, and for this equation to be true, either $x - 4 = 0$ or $x + 7 = 0$. Therefore, $x = 4$ or $x = -7$.

Tip: Pay careful attention to the sign of your solutions when factoring. For example, given $(x - 4)(x + 7) = 0$, it is tempting to say that the solutions are -4 and 7 when in fact the solutions are 4 and -7.

Exercise: Identify the quadratic expression's factors and the equation's solutions.

a. $(x + 5)(x + 2) = 0$

b. $(x - 4)(x + 1) = 0$

c. $(x - 8)(x - 3) = 0$

Exercise: Solve each equation by factoring.

d. $x^2 + 6x - 27 = 0$

e. $x^2 + 12x + 35 = 0$

f. $x^2 - 7x + 8 = -4$

Factoring Quadratics in the Form $ax^2 + bx$

You may also be asked to solve a quadratic equation with a constant term of 0—in other words, one in the form $ax^2 + bx = 0$. You can solve these by factoring out the greatest common factor. Consider the example below:

Example

$4x^2 = 22x$. What is a possible value of x?

$4x^2 = 22x$	
$4x^2 - 22x = 0$	Rearrange the equation so that it's in the form $ax^2 + bx = 0$.
$2x(2x - 11) = 0$	Factor out the greatest common factor.

For the right-hand side to equal 0, one of the factors has to equal 0: $2x = 0$ or $2x - 11 = 0$. Solving each equation, we have $x = 0$ or $x = \frac{11}{2}$.

Exercise: Solve each equation by factoring out the greatest common factor.

a. $8x^2 + 20x = 0$

b. $5x^2 + 8 = 15x + 8$

Special Quadratics

It's also worth paying extra attention to a few "special" quadratic forms that often show up on the SAT:

$$(x + n)^2 = x^2 + 2nx + n^2$$

$$(x + n)(x - n) = x^2 - n^2$$

Exercise: Rewrite each expression by distributing and then combining like terms.

c. $(x - 4)^2$

d. $(x + 8)^2$

e. $(x + 3)(x - 3)$

Exercise: Rewrite each expression in factored form.

f. $x^2 - 36$

g. $x^2 + 14x + 49$

h. $x^2 - 20x + 100$

Practice Questions

4

If $(x + n)^2 = x^2 - 12x + b$, what is the value of b?

A) –36

B) –24

C) 24

D) 36

5

If $25x^2 - 16 = (ax + b)(ax - b)$, where a and b are constants and $b > 0$, which of the following is true?

A) $a = 4$ and $b = 5$

B) $a = 5$ and $b = 4$

C) $a = 16$ and $b = 25$

D) $a = 25$ and $b = 16$

Standard Form and the Quadratic Formula

There are three forms of quadratic equations that you should know for the SAT, and standard form is the most common of the three. (The other two, factored form and vertex form, are discussed in QUA Part 2.)

A quadratic equation in standard form looks like this: $f(x) = ax^2 + bx + c$, where a, b, and c are constants and $a \neq 0$.

Quadratic Formula

You're probably familiar with the quadratic formula. It states that **for an equation in standard form, the solutions are** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. This can be helpful when you are trying to solve an equation that is not easily factored.

Exercise: Solve each equation using the quadratic formula.

a. $x^2 + 5x - 1 = 0$

b. $x^2 + 4 = 6x$

c. $3x^2 = 2 - 8x$

Tip: Desmos is often the quickest way to solve a quadratic equation: simply graph both sides of the equation and find the point(s) of intersection. (See the Problem-Solving with Desmos unit for guidance on using Desmos to solve equations.) At the same time, many challenging quadratics SAT questions build on an understanding of the underlying skills. If you're aiming for a top score, you should know how to solve quadratic equations without Desmos.

Related to the quadratic formula, two more formulas are worth practicing and memorizing: **when a quadratic equation has two real solutions, their sum is** $\frac{-b}{a}$ **and their product is** $\frac{c}{a}$. Using these when applicable may be faster than other methods, including Desmos.

Exercise: Find the sum and product of the solutions.

d. $3x^2 - 18x + 6 = 0$

e. $-3(x + 4)^2 + 10 = 0$

Practice Question

6

$$48x^2 + (48n + m)x + mn = 0$$

In the given equation, m and n are positive constants. The product of the solutions to the given equation is kmn, where k is a constant. What is the value of k?

A) $\frac{1}{48}$

B) $\frac{1}{16}$

C) 1

D) 48

Completing the Square

Completing the square is a method that lets you rewrite a quadratic equation in the form $(x + n)^2 = k$, making the equation easier to solve. It can be helpful when a given equation is not easily factored. Consider the example below:

Solve $x^2 - 14x + 1 = 0$.	
$x^2 - 14x = -1$	Isolate the constant term.
$x^2 - 14x + (-7)^2 = -1 + (-7)^2$ $x^2 - 14x + 49 = -1 + 49$ $x^2 - 14x + 49 = 48$	Add the square of half the coefficient of x. In this example, the coefficient of x is -14; half of it is -7; the square of -7 is 49.
$(x - 7)^2 = 48$	Now, the left-hand side of the equation is in the common quadratic form $x^2 + 2nx + n^2$, which equals $(x + n)^2$.
$x - 7 = \pm\sqrt{48}$ $x = 7 \pm \sqrt{48}$ or $7 \pm 4\sqrt{3}$	Isolate x by using appropriate inverse operations.

Tip: It is easy to forget to balance your equation when completing the square: make sure that, once you have added a constant to the side with the quadratic expression, you also add that same constant to the other side.

Exercise: Solve each equation by completing the square.

a. $x^2 + 6x - 5 = 0$

b. $x^2 - 4x - 8 = 0$

c. $x^2 - 6x - 16 = 0$

Practice Questions

7

$$x^2 - 2x - 5 = 0$$

One solution to the given equation can be written as $1 + \sqrt{k}$, where k is a constant. What is the value of k?

A) 6

B) 7

C) 12

D) 24

8

$$4x^2 - 40x + 4 = 0$$

Which of the following is a solution to the equation above?

A) $-2\sqrt{6}$

B) $5 - 4\sqrt{6}$

C) $5 - 2\sqrt{6}$

D) $10 - 2\sqrt{6}$

Summary

In this unit, you learned that:

- ✔ Standard form is the most common form of quadratic equation on the SAT. It looks like this: $f(x) = ax^2 + bx + c$, where a, b, and c are constants and $a \neq 0$.
- ✔ The quadratic formula states that the solutions for an equation in the form $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. You should memorize this formula.
- ✔ Factoring is a key skill for many SAT questions on quadratics.
- ✔ It is helpful to be able to recognize "special" quadratics. $(x + n)^2 = x^2 + 2nx + n^2$, and $(x + n)(x - n) = x^2 - n^2$, where n is a constant.
- ✔ Completing the square lets you rewrite a quadratic equation in the form $(x + n)^2 = k$, making the equation easier to solve.

Answers

1. B
2. B
3. 8
4. D
5. B
6. A
7. A
8. C

Exercises

p. 4: **a**. Factors: $(x + 5)$ and $(x + 2)$; solutions: –5 and –2, **b**. Factors: $(x - 4)$ and $(x + 1)$; solutions: –1 and 4, **c.** Factors: $(x - 8)$ and $(x - 3)$; solutions: 3 and 8, **d.** –9 and 3, **e.** –7 and –5, **f.** 3 and 4

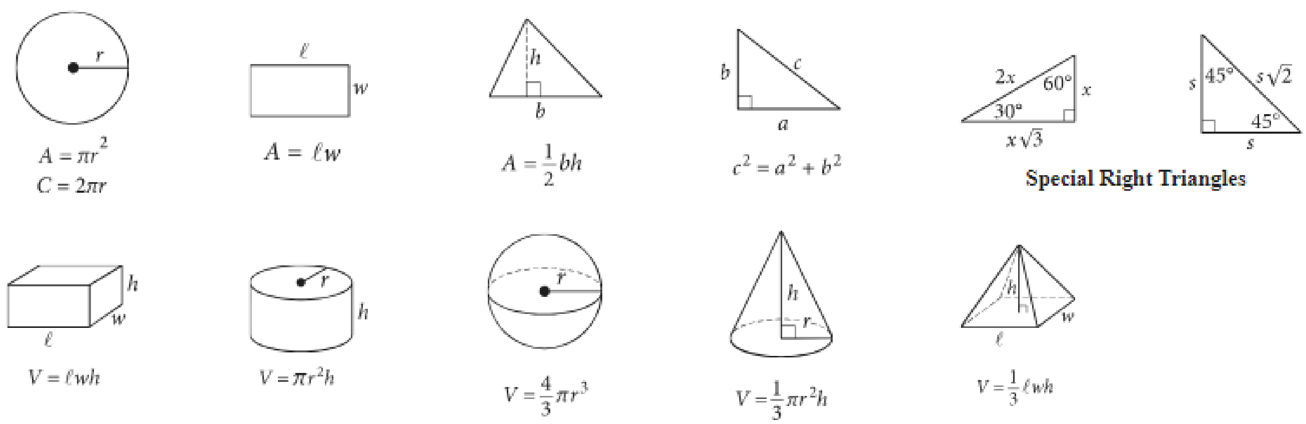
p. 5: **a.** $8x^2 + 20x = 4x(2x + 5) = 0$, so $4x = 0$ or $2x + 5 = 0$; the solutions are $-\frac{5}{2}$ and 0, **b.** $5x^2 + 8 = 15x + 8 \rightarrow 5x^2 - 15x = 0 \rightarrow 5x(x - 3) = 0$, so $5x = 0$ or $x - 3 = 0$; the solutions are 0 and 3, **c**. $x^2 - 8x + 16$, **d**. $x^2 + 16x + 64$, **e**. $x^2 - 9$, **f**. $(x + 6)(x - 6)$, **g.** $(x + 7)^2$, **h.** $(x - 10)^2$

p. 6: **a.** $-\frac{5}{2} \pm \frac{\sqrt{29}}{2}$, **b.** $3 \pm \sqrt{5}$, **c.** $-\frac{4}{3} \pm \frac{\sqrt{22}}{3}$, **d.** Sum is 6; product is 2, **e.** Sum is –8; product is $\frac{38}{3}$

p. 8: **a**. $-3 \pm \sqrt{14}$, **b**. $2 \pm 2\sqrt{3}$, **c.** –2, 8

Homework: Quadratic Equations & Functions, Part 1:

Complete the following homework exercises to practice the strategies and content you studied in the previous chapter.



The number of degrees of arc in a circle is 360.

The number of radians of arc in a circle is 2π.

The sum of the measures in degrees of the angles of a triangle is 180.

1

$$h(x) = -2x^2 + 11$$

Which table gives three values of x and their corresponding values of $h(x)$ for the given function h?

A)

x	$h(x)$
1	15
2	27
3	47

B)

x	$h(x)$
1	9
2	3
3	-7

C)

x	$h(x)$
1	9
2	7
3	5

D)

x	$h(x)$
1	13
2	19
3	29

2

The function f is defined by $f(x) = x^2 - 8$. For which value of x is $f(x) = 28$?

A) –8

B) 6

C) 20

D) 36

3

Which expression is equivalent to $x^2 + 2x - 24$?

A) $(x - 8)(x - 3)$

B) $(x + 8)(x - 3)$

C) $(x + 8)(x - 6)$

D) $(x + 6)(x - 4)$

4

Which expression is equivalent to $x^2 + x - 42$?

A) $(x - 3)(x + 14)$

B) $(x - 6)(x + 7)$

C) $(x - 7)(x + 6)$

D) $(x - 14)(x + 3)$

5

If a and b are positive numbers, which of the following is equivalent to $\frac{\sqrt{(a+b)^5}}{\sqrt{a+b}}$?

A) $a + b$

B) $a^2 + b^2$

C) $a^2 + 2ab + b^2$

D) a^2b^2

6

$$x^2 + qx + 28 = 0$$

In the given equation, q is a positive constant. The solutions to the given equation are integers. Which of the following is a possible value of q?

I. 11

II. 27

III. 29

A) I only

B) II only

C) I and III only

D) I, II, and III

7

$$\frac{x^2-b}{x-a}$$

In the expression above, a and b are positive integers. If the expression is equivalent to $x + a$, which of the following could be the value of b?

A) 10

B) 12

C) 14

D) 16

8

$$5x^2 - 10x - 15 = 0$$

What is the positive solution to the given equation?

9

$$x^2 - 3x - 19 = x + 2$$

What is the negative solution to the given equation?

10

The expression $(2x + 21)(17x - 5)$ is equivalent to the expression $ax^2 + bx + c$, where a, b, and c are constants. What is the value of b?

11

$$42x^2 + (42 + q)x + qr = 0$$

In the given equation, q and r are positive constants. The sum of the solutions to the given equation is 2. What is the value of q?

12

$$-2x^2 - 6x = 8x - 10$$

What is the product of the solutions to the given equation?

13

$$x^2 - 6x =- 2$$

One of the solutions to the given equation can be written as $a + \sqrt{b}$, where a and b are constants. What is the value of $a + b$?

14

$$2x^2 - 4 =- 6x - 1$$

Which of the following is a solution to the equation above?

A) $\frac{-3-\sqrt{15}}{2}$

B) $-\frac{3}{2} - \sqrt{3}$

C) $3 + \frac{\sqrt{15}}{2}$

D) $3 + \sqrt{3}$

15

$$\sqrt{(x + 2)^2} = \sqrt{4 - 2x}$$

What is the smallest solution to the given equation?

16

Which of the following expressions is a factor of $4x^2 - 20x - 56$?

I. $4x - 8$

II. $x + 7$

A) I only

B) II only

C) I and II

D) Neither I nor II

17

$$x^2 + bx + c = 0$$

In the given equation, b and c are constants. If $-b + \sqrt{b^2 - 4ac} = 24$ and $-b - \sqrt{b^2 - 4ac} = 4$, what is one possible value of x?

Answers

1. B
2. B
3. D
4. B
5. C
6. C
7. D
8. 3
9. –3
10.
11. 347
12. –126
13. –5
14. 10
15. A
16. –6
17. D
18. 2 or 12